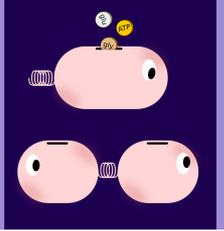


Economic Principles in Cell Biology

Paris, July 10-14, 2023



Cell division coordination

Mattia Corigliano, SPCG Group @ IFOM

Book chapter authors:

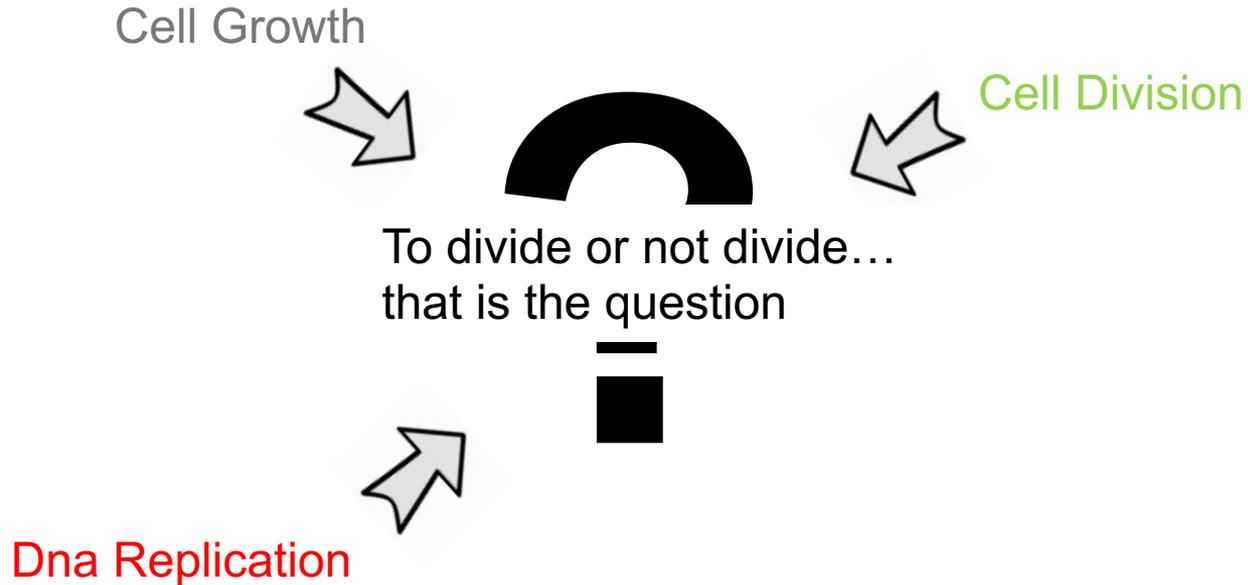
Mattia Corigliano, IFOM

Jacopo Grilli, ICTP

Gabriele Micali, Humanitas

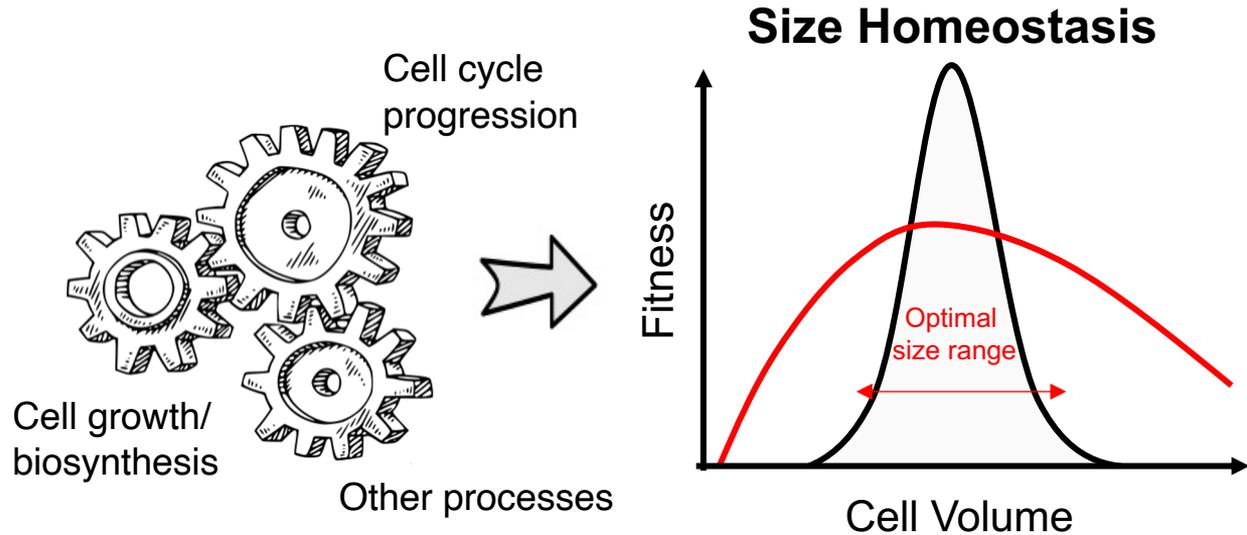
Marco Cosentino-Lagomarsino, IFOM & University of Milan

Cell division control/coordination



Why cell division control/coordination?

Because coordination of cell division, growth and other processes allow for cellular homeostasis and optimal functioning



Zatulovskiy, E and Skotheim, JM, Trends in Genetics, May 2020, Vol. 36, No. 5
Neurohr, GE and Amon, A, Trends in Cell Biology, March 2020, Vol. 30, No. 3
Xie, S., et al, Annual review of cell and developmental biology, May 2022.

Because it's fun



Disclaimer

This lecture will focus on the bacterium *E. coli* and “single-cell” data

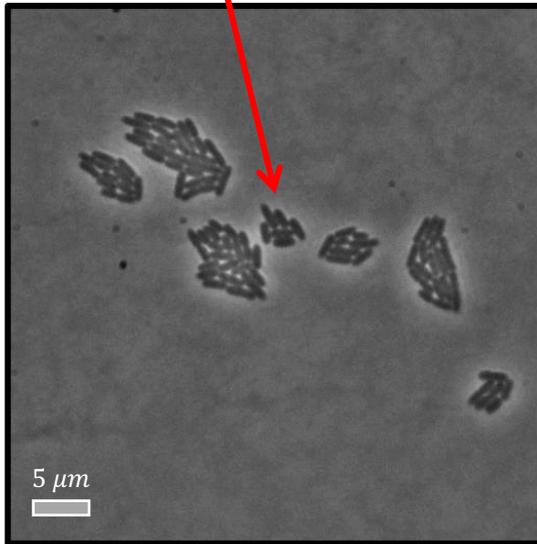
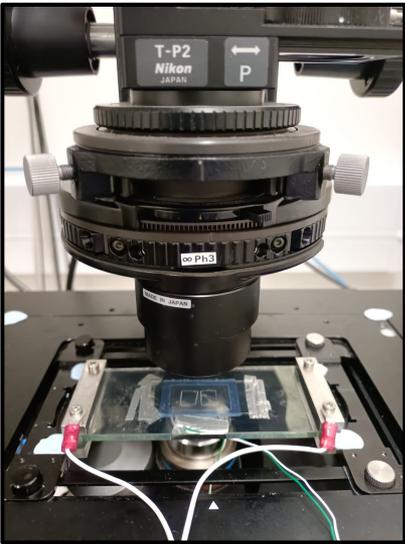
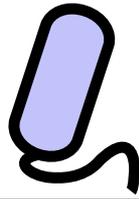


Image source: Mattia Corigliano



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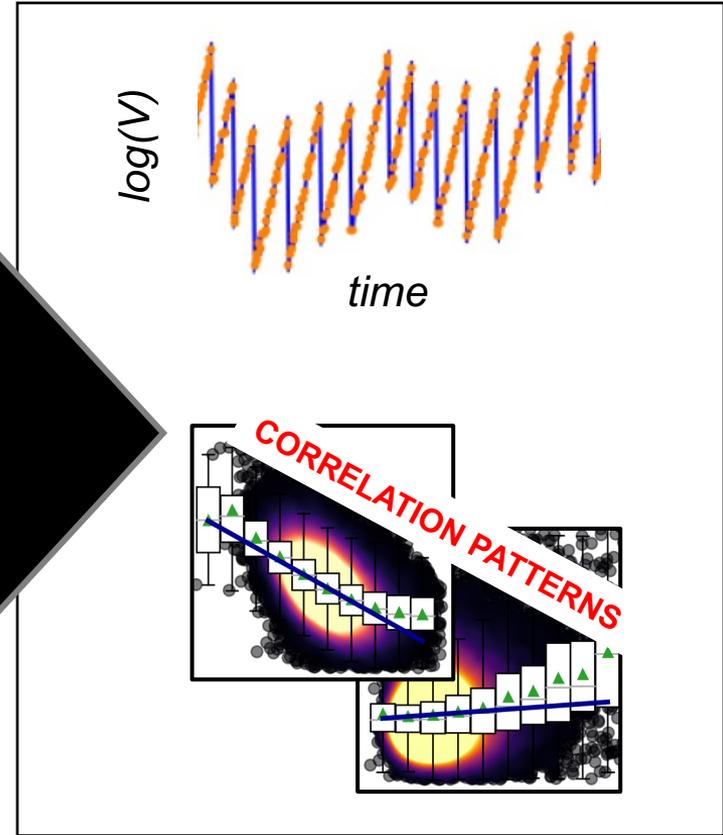
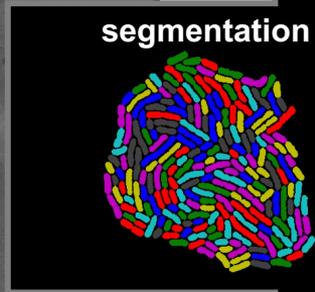
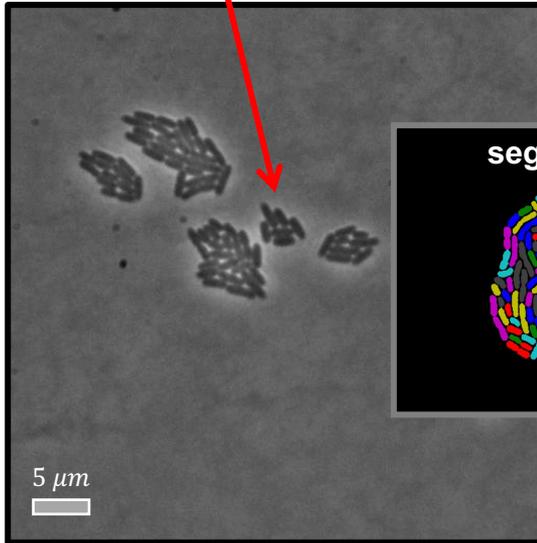
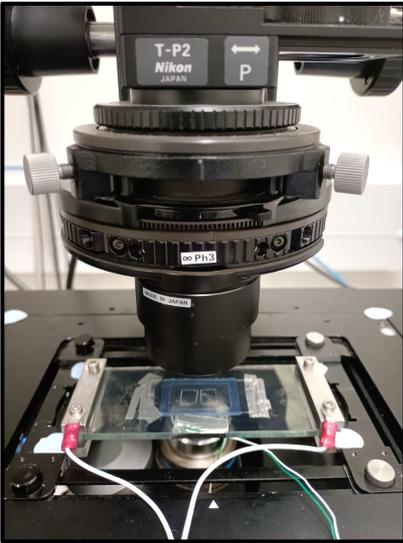
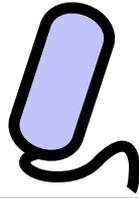
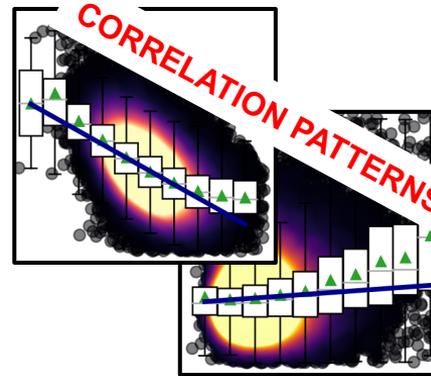


Image source: Mattia Corigliano



Book chapter question (I)

How single-cell correlation patterns can be used to understand cell-division behaviors?



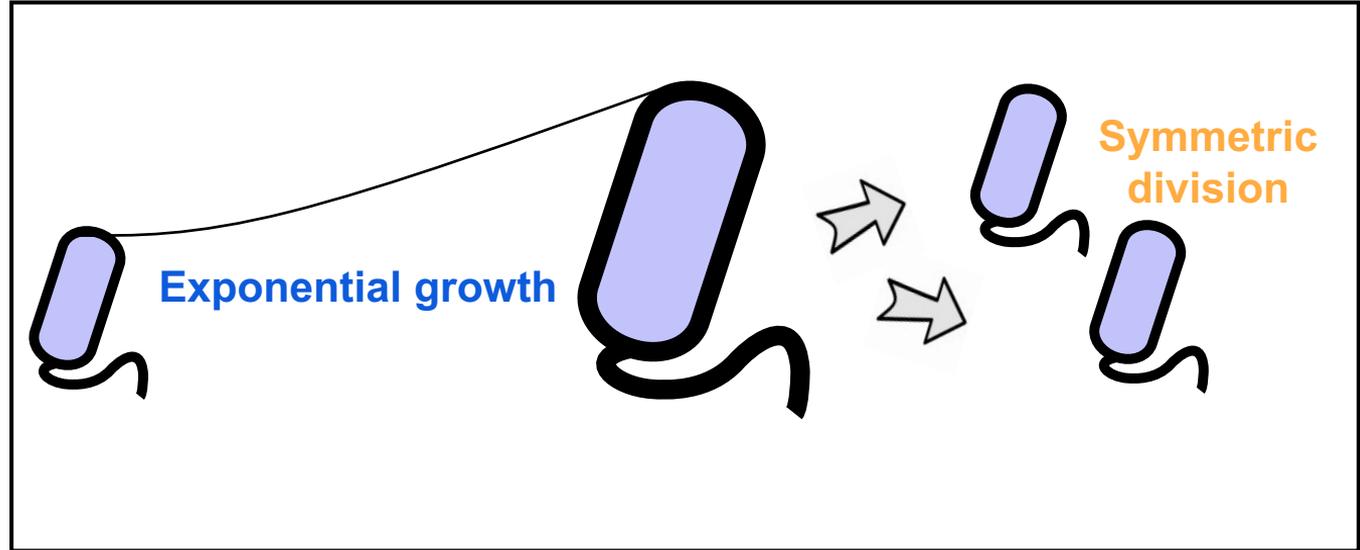
Quantitative approaches to cell division control

1. Linear response approach
2. Hazard rate approach (if we have time)

(Sections 12.2, 12.3)



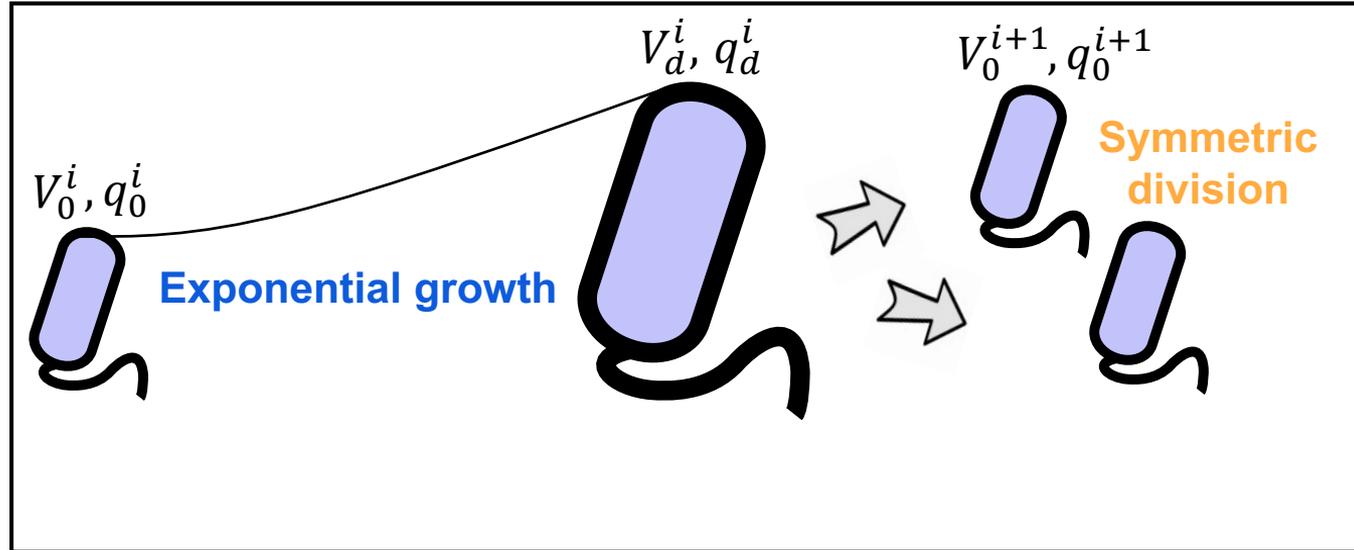
Known facts of *E.coli* growth and division



Known facts of *E.coli* growth and division

Notation

V_0^i	Volume at birth
V_d^i	Volume at division
q_0^i	Log. Volume at birth
q_d^i	Log. Volume at division
α^i	Growth rate
τ_d^i	Division time
G^i	Elongation



Def.

$$q_0^i \equiv \log V_0^i / V^*$$

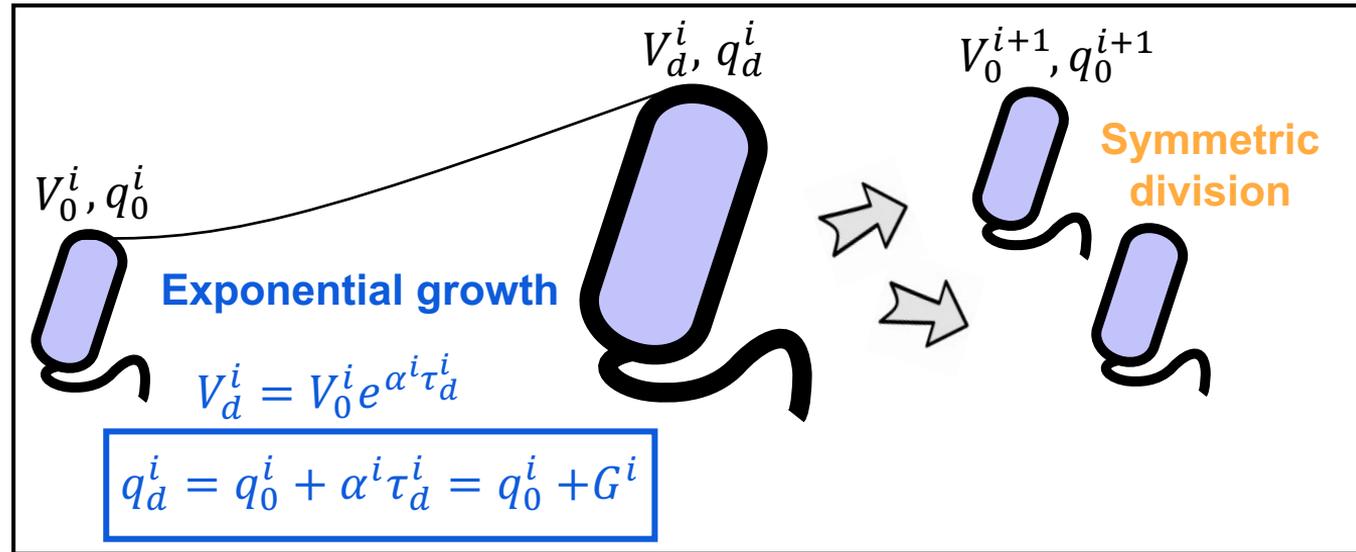
$$G^i \equiv \alpha^i \tau_d^i = q_d^i - q_0^i$$



Known facts of *E.coli* growth and division

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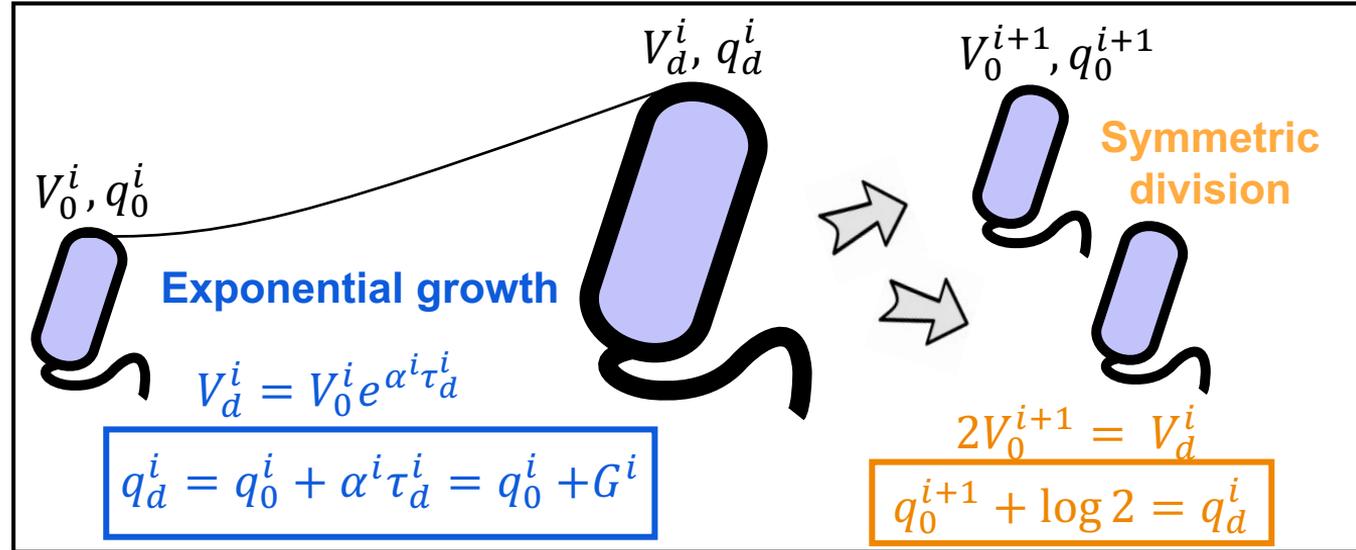
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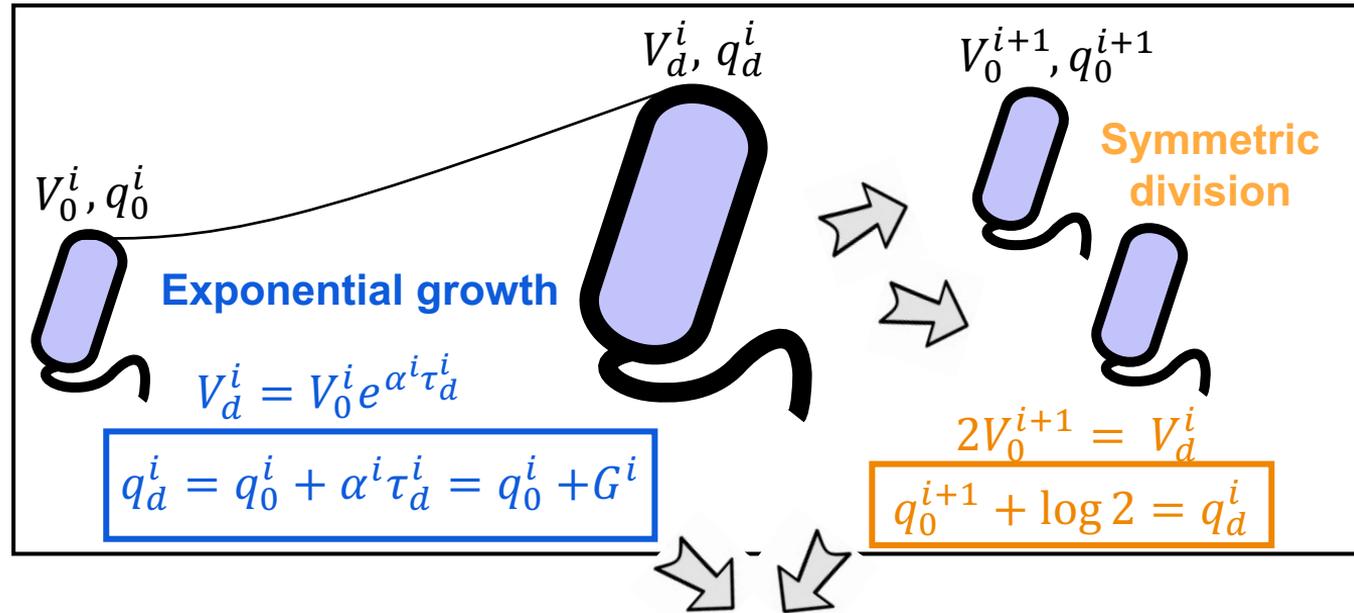
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$$G^i \equiv \alpha^i \tau_d^i = q_d^i - q_0^i$$

$$q_0^{i+1} = q_0^i + \delta G^i = q_0^i + \delta(\alpha^i \tau_d^i)$$

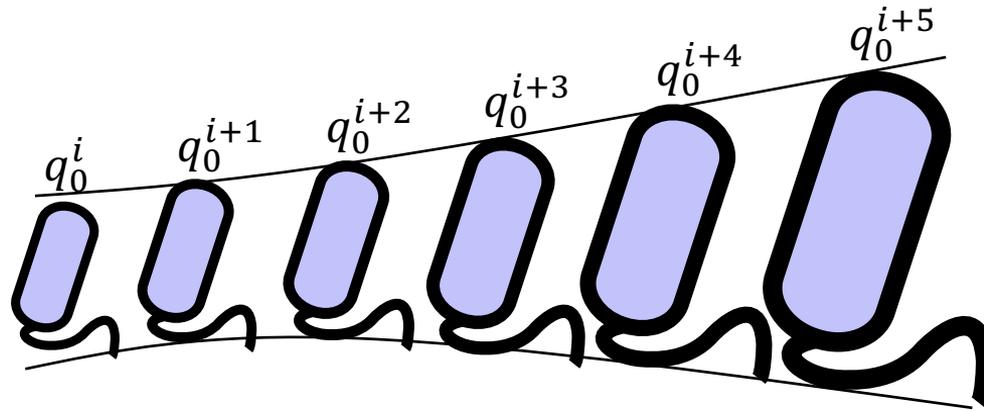


Division *per se* does not guarantee cell size homeostasis

Mechanisms must be in place to control cell size

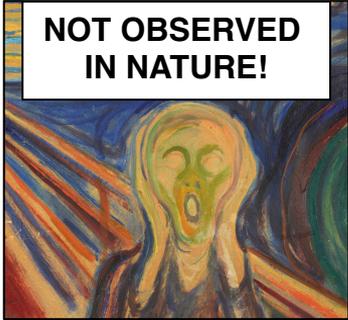
$$q_0^{i+1} = q_0^i + \delta G^i = q_0^i + \delta(\alpha^i \tau_d^i)$$

Random noise \rightarrow Random walk



Amir A, Physical Review Letters **112** (2014)

NOT OBSERVED
IN NATURE!

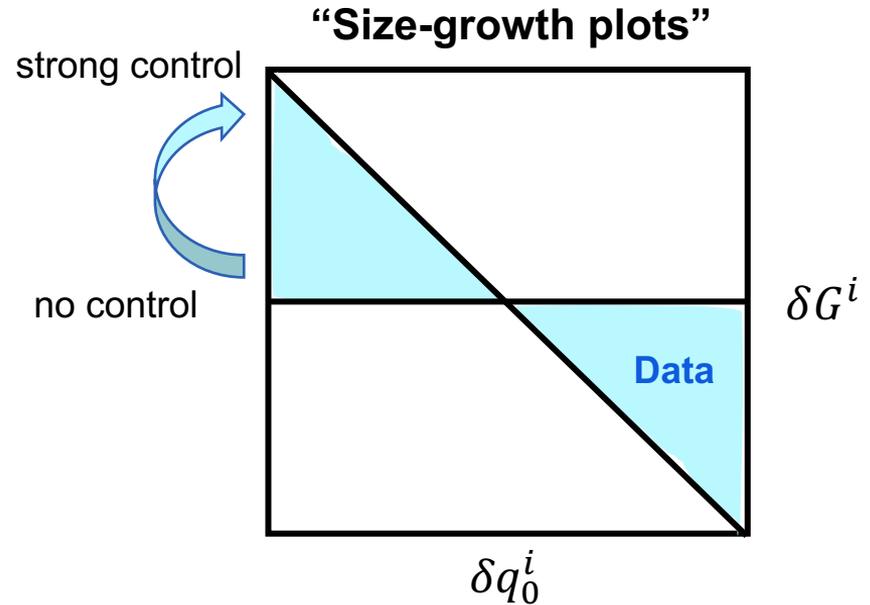
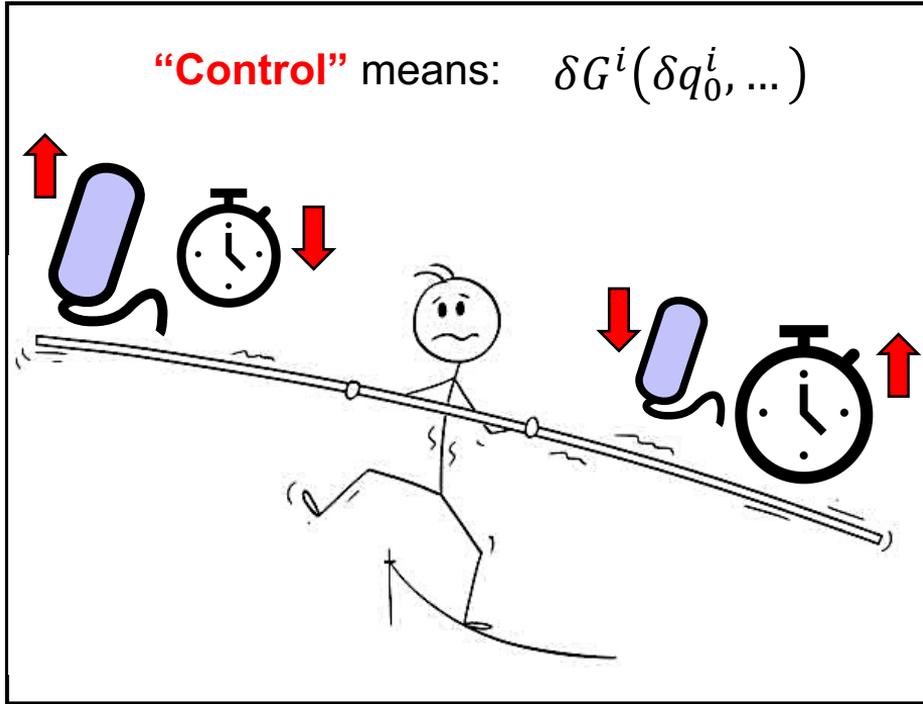


Exercise: what if cell growth is linear and /or cell division is asymmetric?



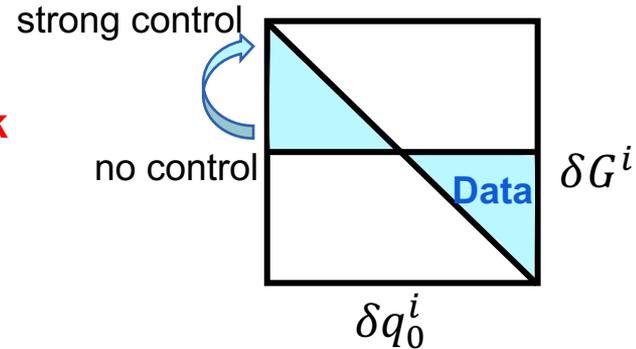
Mechanisms must be in place to control cell size

“Control” means: $\delta G^i(\delta q_0^i, \dots)$



Book chapter question (I bis)

Can we set up a quantitative framework within which making sense of these plots?



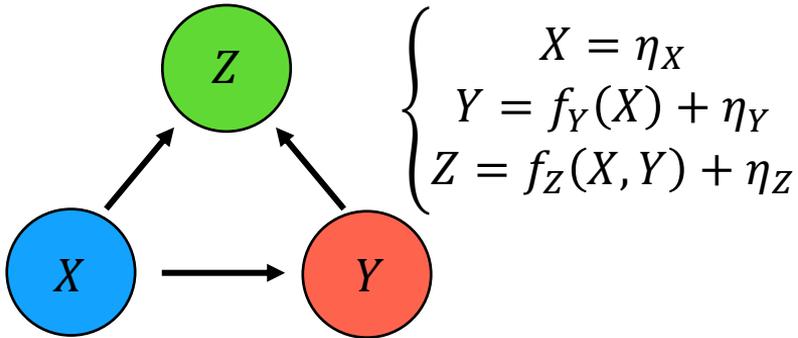
Quantitative approaches to cell division control

1. Linear response approach
2. Hazard rate approach (if we have time)

(Sections 12.2, 12.3)



Linear-response 101



Central assumption:

$$f_Y(X) \approx \langle Y \rangle - \lambda_{YX} \delta X$$

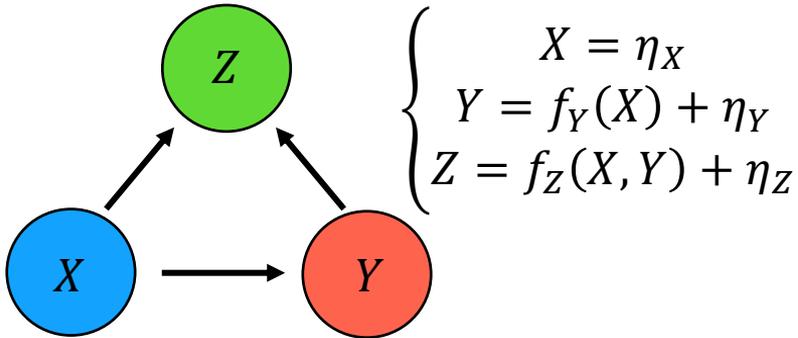
$$f_Z(X, Y) \approx \langle Z \rangle - \lambda_{ZX} \delta X - \lambda_{ZY} \delta Y$$

$$\lambda_{ab} \equiv -\frac{\partial f_a}{\partial b} (\langle b \rangle, \dots) \equiv \Lambda_{ab} \frac{\sigma_a}{\sigma_b}$$

Control parameters



Linear-response 101



Central assumption:

$$f_Y(X) \approx \langle Y \rangle - \lambda_{YX} \delta X$$

$$f_Z(X, Y) \approx \langle Z \rangle - \lambda_{ZX} \delta X - \lambda_{ZY} \delta Y$$

$$\lambda_{ab} \equiv -\frac{\partial f_a}{\partial b} (\langle b \rangle, \dots) \equiv \Lambda_{ab} \frac{\sigma_a}{\sigma_b}$$

Control parameters

Linear response

$$\begin{cases} \frac{\delta Y}{\sigma_Y} = -\Lambda_{YX} \frac{\delta X}{\sigma_X} + \eta_Y \\ \frac{\delta Z}{\sigma_Z} = -\Lambda_{ZX} \frac{\delta X}{\sigma_X} - \Lambda_{ZY} \frac{\delta Y}{\sigma_Y} + \eta_Z \end{cases}$$

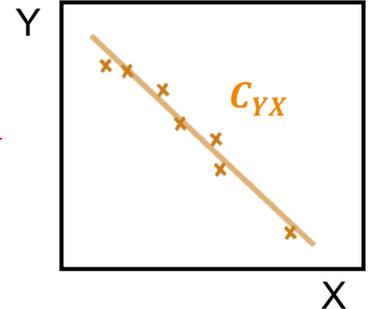


Correlation patterns

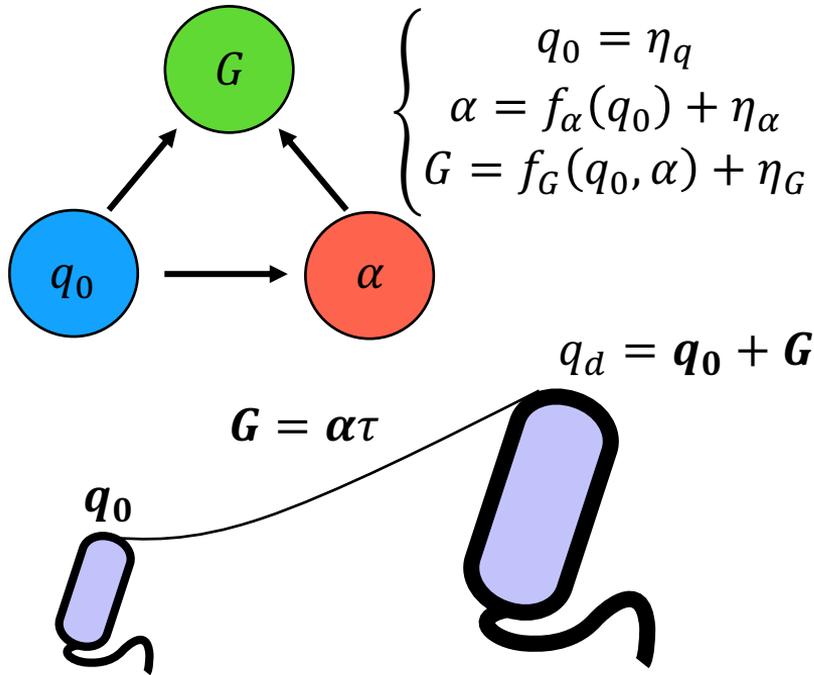
$$C_{YX} = -\Lambda_{YX}$$

$$C_{ZX} = -\Lambda_{ZX} + \Lambda_{ZY} \Lambda_{YX}$$

$$C_{ZY} = \Lambda_{ZX} \Lambda_{XY} - \Lambda_{ZY}$$



E.coli linear-response model of cell division control

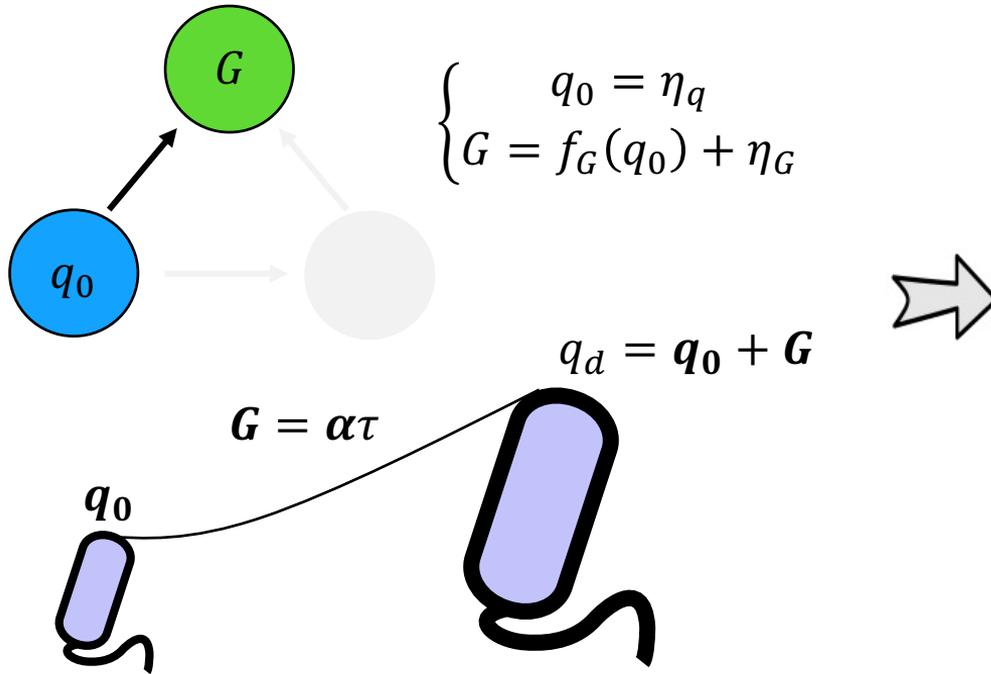


Grilli J., et al., *Frontiers in Microbiology* 9 (2018)

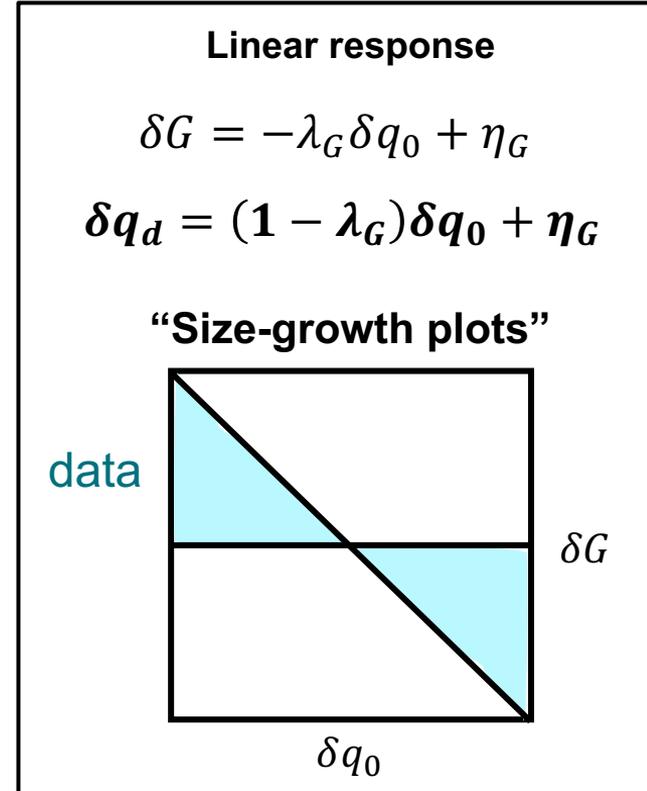


E.coli linear-response model of cell division control

Even simpler because growth rate fluctuations are usually neglected



Grilli J., *et al.*, *Frontiers in Microbiology* 9 (2018)



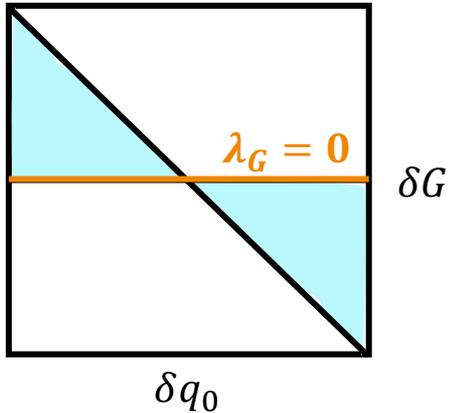
Control mechanisms

Linear response

$$\delta G = -\lambda_G \delta q_0 + \eta_G$$

$$\delta q_d = (1 - \lambda_G) \delta q_0 + \eta_G$$

“Size-growth plots”



Timer: $\delta G^i = 0$

$$\delta \tau_d^i = 0 \rightarrow \tau_d^i = \langle \tau_d \rangle$$

$$\delta q_0^{i+1} = \delta q_0^i$$

Uncontrolled fluctuations



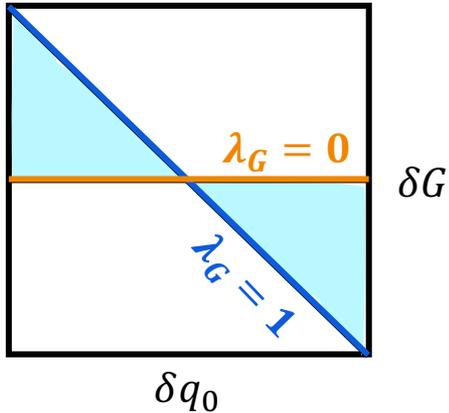
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$$\delta q_0^{i+1} = \delta q_0^i$$

Uncontrolled fluctuations

Sizer: $\delta G^i = -\delta q_0^i$

$$\tau_d^i = \langle \tau_d \rangle - \alpha^{-1} \delta q_0^i$$

$$\delta q_0^{i+1} = 0 \rightarrow q_d^i = \langle q_d \rangle$$

Fluctuations controlled in one cell cycle



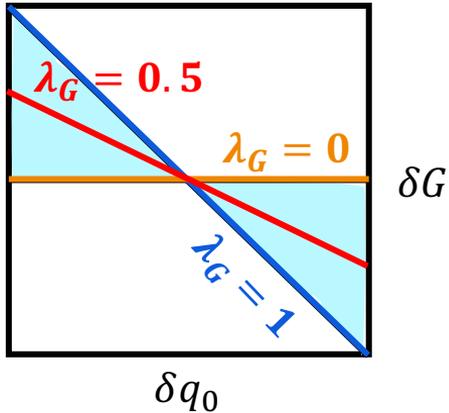
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Amir A, Physical Review Letters 112 (2014)

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Uncontrolled fluctuations

Sizer: $\delta G^i = -\delta q_0^i$

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Fluctuations controlled in one cell cycle

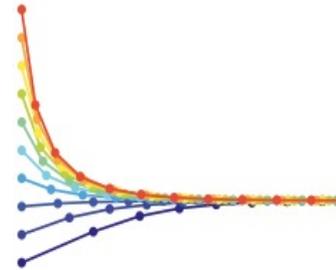
Adder: $\delta G^i = -\delta q_0^i / 2$

$$\tau_d^i = \langle \tau_d \rangle - \delta q_0^i / 2\alpha$$

$$\delta q_0^{i+1} = \delta q_0^i / 2$$

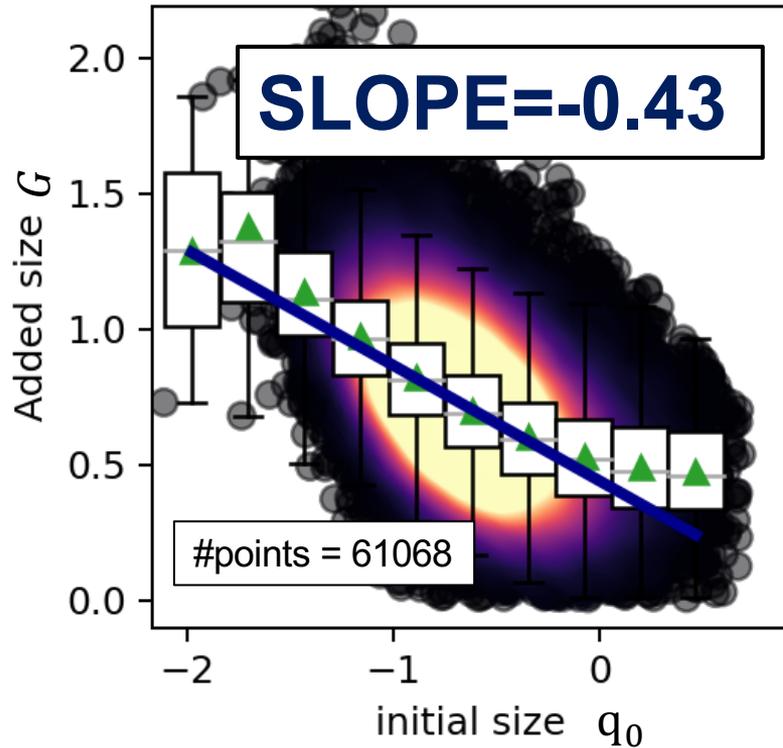
$$* V_d^i = V_0^i + \Delta *$$

Exponentially controlled fluctuations



Evidence of adder-like correlations

E.Coli cells implement the adder strategy

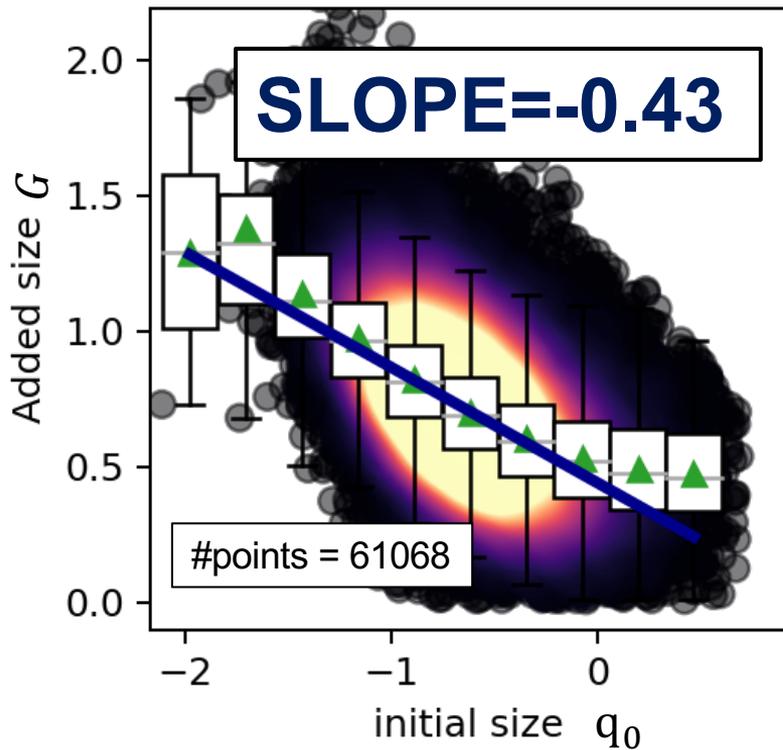


[Data from Mattia's experiments]

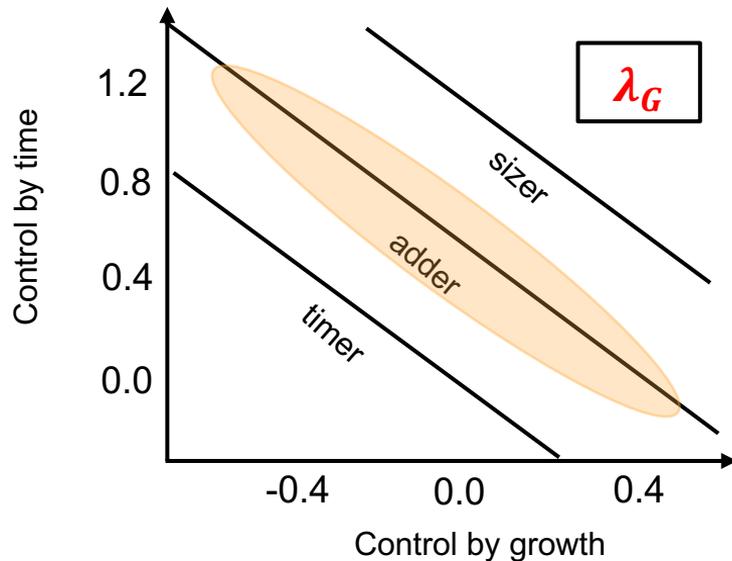


Evidence of adder-like correlations

E.Coli cells implement the adder strategy



$$\delta G^i = \langle \alpha \rangle \delta \tau_d^i + \langle \tau_d \rangle \delta \alpha^i$$



Data from different species
in different conditions

[Adapted from Cadart et al. Nat Commun. (2018)]

[Data from Mattia's experiments]



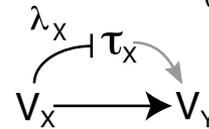
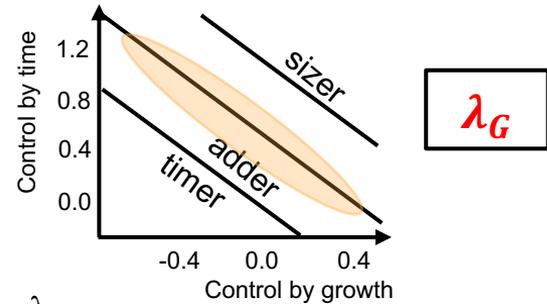
Features of linear response framework

A general solvable framework which:

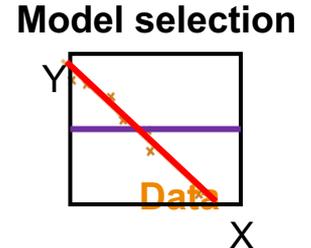
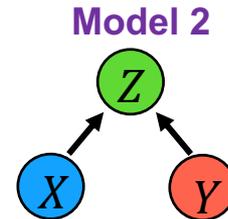
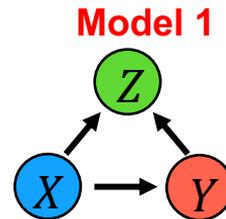
□ Allows us to interpret and compare control strategies of different species in different conditions

□ Can be easily modified to describe more refined cell cycle models (cell cycle subperiods)

□ Relates correlation patterns in data to couplings/mechanisms in models:
 - propose mechanisms
 - select scenarios



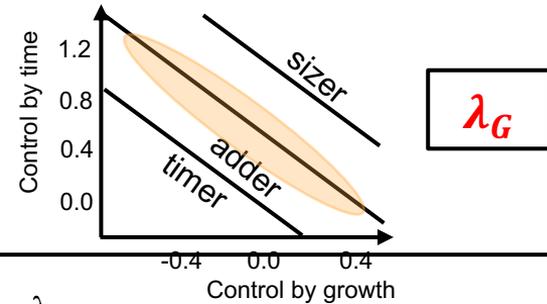
$$\delta q_Y^i = (1 - \lambda_Y) \delta q_X^i + \eta^i$$



Features of linear response framework

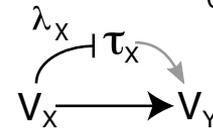
A general solvable framework which:

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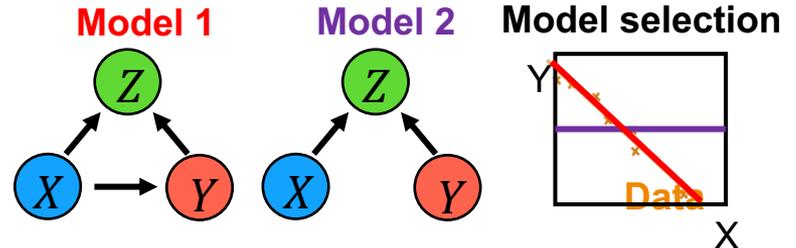
Next

- Can be easily modified to describe more refined cell cycle models (cell cycle subperiods)



$$\delta q_Y^i = (1 - \lambda_Y) \delta q_X^i + \eta^i$$

- Relates correlation patterns in data to couplings/mechanisms in models:
 - propose mechanisms
 - select scenarios



Book chapter question (II)

**What sets the decision to divide?
What is the rate-limiting process
setting cell division?**

**Coordination of cell division with different
cell-cycle processes**

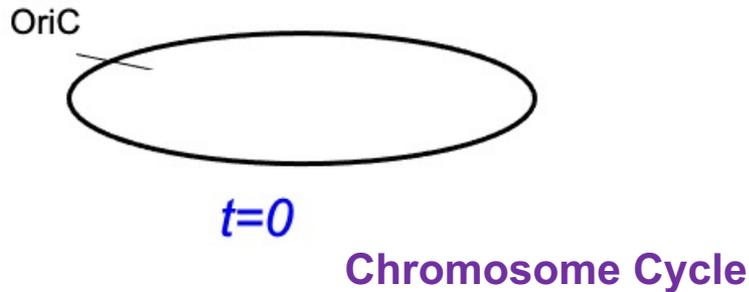
(Sections 12.4)



What is the rate-limiting process setting cell division?

'Traditional answer' *based on population averages*
(Replication + segregation) is rate-limiting for division

(*Schaechter et al 1958, Cooper & Helmstetter 1968, Donachie 1969*)



**Division a “constant” time C+D
after initiation of DNA replication**

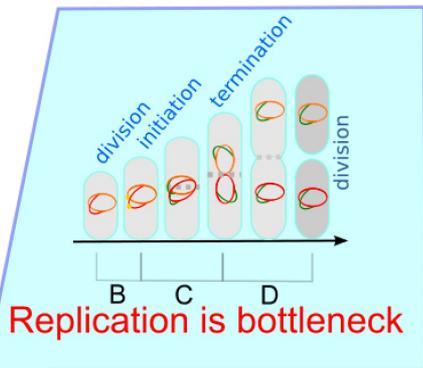
True for single cells ?



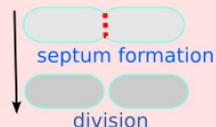
What is the rate-limiting process setting cell division?

For single cells two opposite views were proposed

Wallden *et al.*
Cell (2016)
Ho & Amir
Front. Microb.
(2015)



Replication is
never bottleneck



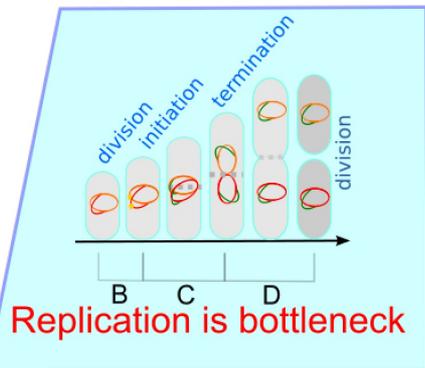
Harris & Theriot
Cell (2016),
TIM (2018)



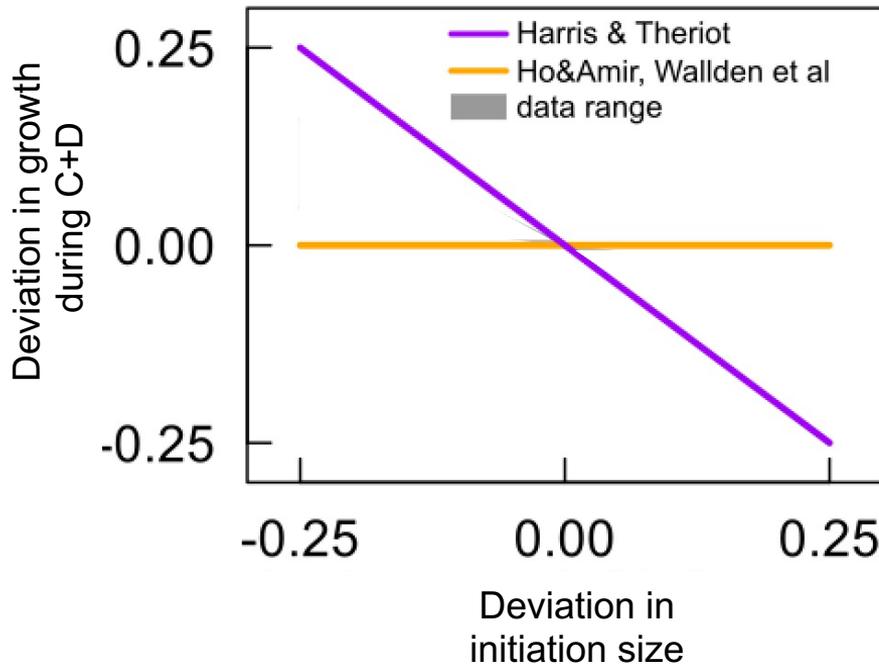
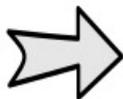
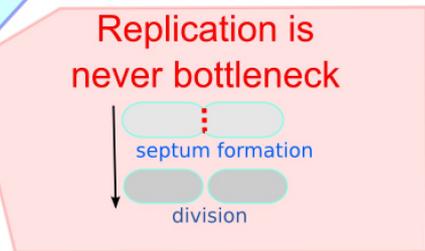
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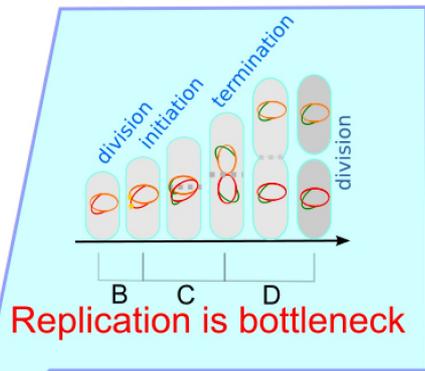


Harris & Theriot
Cell (2016),
TIM (2018)



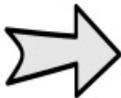
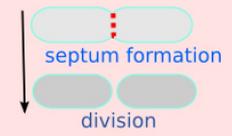
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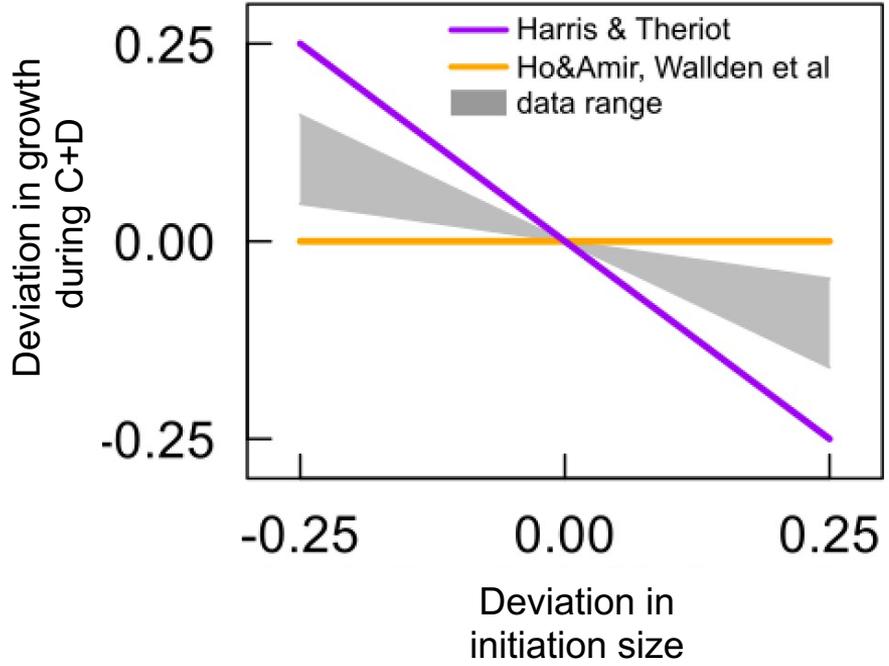


Replication is never bottleneck

Harris & Theriot
Cell (2016),
TIM (2018)

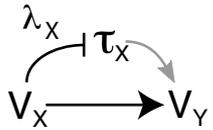


They both fail to reproduce relevant correlation patterns (but they didn't know)!



Generalization of replication-limiting models

Recipe



$$\delta q_Y^i = (1 - \lambda_Y) \delta q_X^i + \text{noise}$$

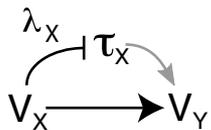
$$1 - \lambda_Y \equiv \widetilde{\lambda}_Y$$



Generalization of replication-limiting models

"Wallden et al. *Cell* (2016)"

Recipe

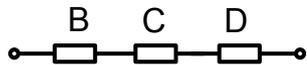


$$\delta q_Y^i = (1 - \lambda_Y) \delta q_X^i + \text{noise}$$

$$1 - \lambda_Y \equiv \widetilde{\lambda}_Y$$

intervals in series

"BCD" models



$$\delta q_B^i = \widetilde{\lambda}_B \delta q_0^i + \eta_B^i$$

$$\delta q_C^i = \widetilde{\lambda}_C \delta q_B^i + \eta_C^i$$

$$\delta q_D^i = \widetilde{\lambda}_D \delta q_C^i + \eta_D^i$$



$$\widetilde{\lambda}_G = \widetilde{\lambda}_{C+D} \widetilde{\lambda}_B$$

$$\widetilde{\lambda}_I = \widetilde{\lambda}_G$$

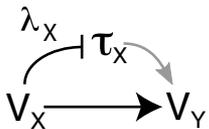


Generalization of replication-limiting models

"Wallden et al. *Cell* (2016)"

Ho & Amir *Front. Microbiol.* (2015)

Recipe

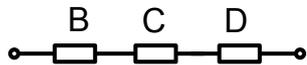


$$\delta q_Y^i = (1 - \lambda_Y) \delta q_X^i + \text{noise}$$

$$1 - \lambda_Y \equiv \widetilde{\lambda}_Y$$

intervals in series

"BCD" models



$$\delta q_B^i = \widetilde{\lambda}_B \delta q_0^i + \eta_B^i$$

$$\delta q_C^i = \widetilde{\lambda}_C \delta q_B^i + \eta_C^i$$

$$\delta q_D^i = \widetilde{\lambda}_D \delta q_C^i + \eta_D^i$$

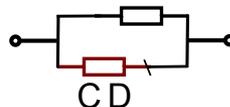


$$\widetilde{\lambda}_G = \widetilde{\lambda}_{C+D} \widetilde{\lambda}_B$$

$$\widetilde{\lambda}_I = \widetilde{\lambda}_G$$

intervals in parallel

"ICD" models



$$\delta q_B^i = \widetilde{\lambda}_I \delta q_B^{i-1} + \eta_B^i$$

$$\delta q_C^i = \widetilde{\lambda}_C \delta q_B^i + \eta_C^i$$

$$\delta q_D^i = \widetilde{\lambda}_D \delta q_C^i + \eta_D^i$$



$$\widetilde{\lambda}_G = \widetilde{\lambda}_{C+D} \widetilde{\lambda}_B$$

$$\widetilde{\lambda}_I = \widetilde{\lambda}_{C+D} \widetilde{\lambda}_B$$



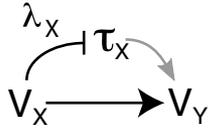
Generalization of replication-limiting models

"Wallden et al. *Cell* (2016)"

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Micali G et al., *Cell reports* **25** (2018)

Recipe

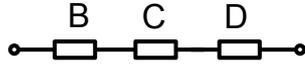


$$\delta q_Y^i = (1 - \lambda_Y) \delta q_X^i + \text{noise}$$

$$1 - \lambda_Y \equiv \widetilde{\lambda}_Y$$

intervals in series

"BCD" models



$$\delta q_B^i = \widetilde{\lambda}_B \delta q_0^i + \eta_B^i$$

$$\delta q_C^i = \widetilde{\lambda}_C \delta q_B^i + \eta_C^i$$

$$\delta q_D^i = \widetilde{\lambda}_D \delta q_C^i + \eta_D^i$$

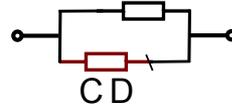


$$\widetilde{\lambda}_G = \widetilde{\lambda}_{C+D} \widetilde{\lambda}_B$$

$$\widetilde{\lambda}_I = \widetilde{\lambda}_G$$

intervals in parallel

"ICD" models



$$\delta q_B^i = \widetilde{\lambda}_I \delta q_B^{i-1} + \eta_B^i$$

$$\delta q_C^i = \widetilde{\lambda}_C \delta q_B^i + \eta_C^i$$

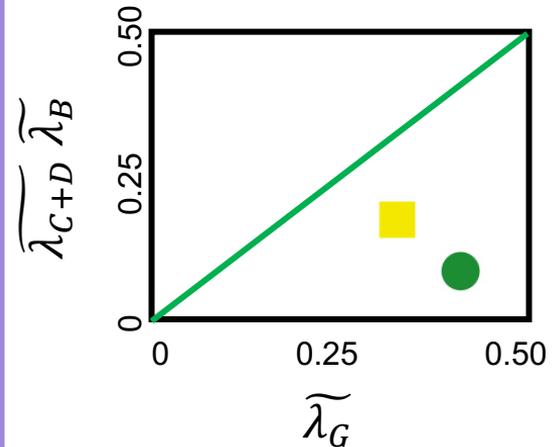
$$\delta q_D^i = \widetilde{\lambda}_D \delta q_C^i + \eta_D^i$$



$$\widetilde{\lambda}_G = \widetilde{\lambda}_{C+D} \widetilde{\lambda}_B$$

$$\widetilde{\lambda}_I = \widetilde{\lambda}_{C+D} \widetilde{\lambda}_B$$

→ Replication is not always rate-limiting for cell division



- Wallden SeqA slow
- Adiciptaningrum
- BCD/ICD (replication limiting)



The concurrent-cycles model

Micali G et al., Cell reports **25** (2018)

Micali G et al., Science Advance **4** (2018)

Replication agnostic process



$$q_H^i = q_H^* + \tilde{\lambda}_H(q_0^i - q_0^*)$$

Replication centric process



$$q_R^i = q_R^* + \delta q_I^i$$

$$\delta q_I^{i+1} = \tilde{\lambda}_I \delta q_I^i$$

$$q_d^i = \max(q_H^i, q_R^i)$$

$$q_d^i \approx q_H^i \sigma^i + q_R^i (1 - \sigma^i)$$



p_H probability of inter-division process being bottleneck

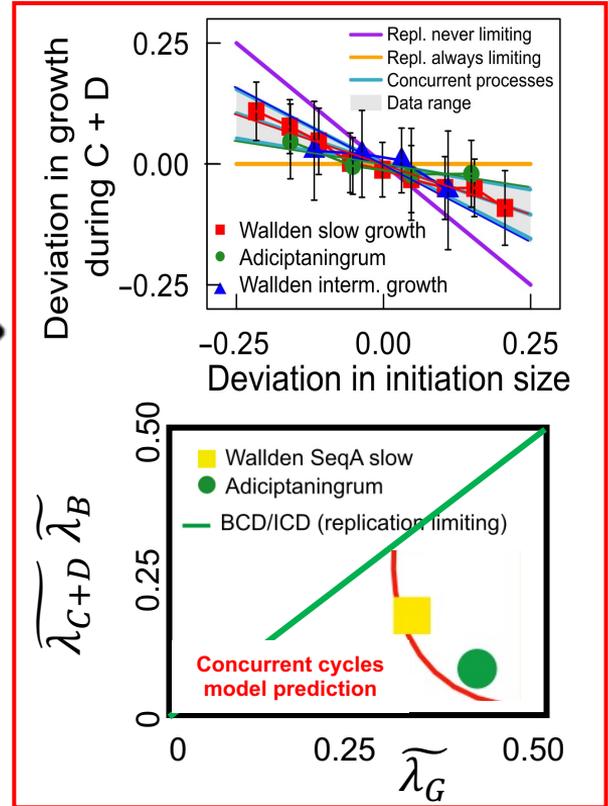
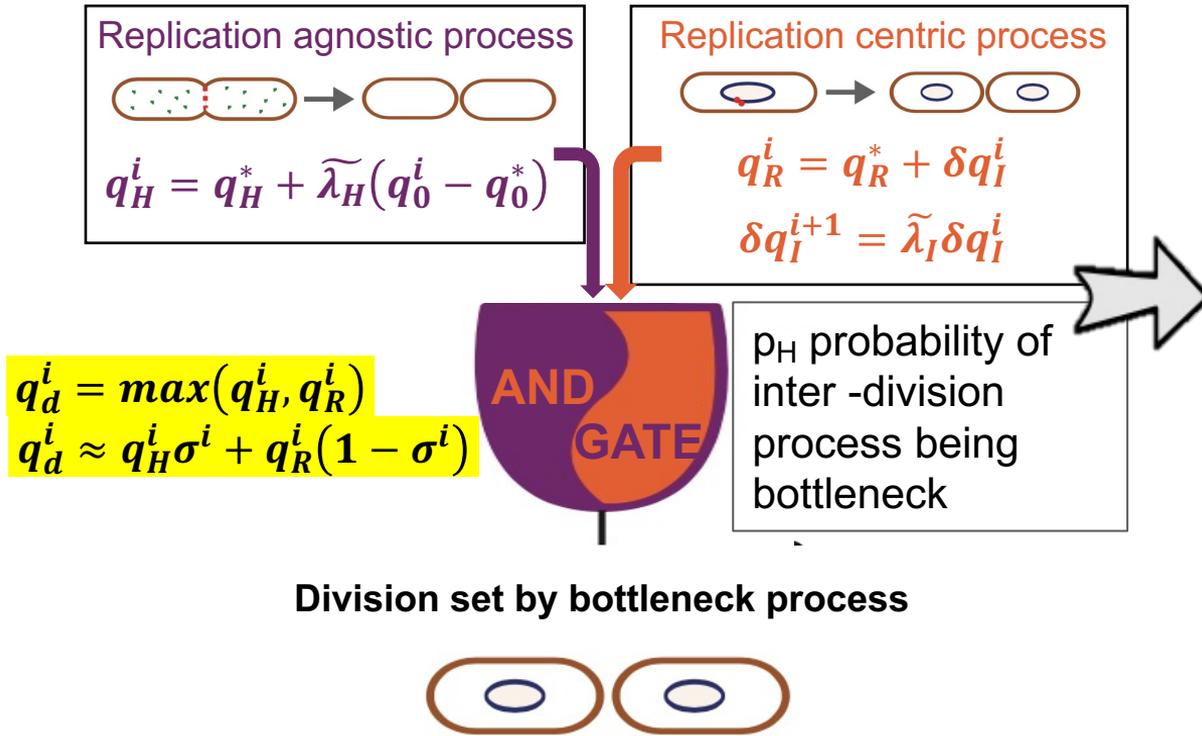
Division set by bottleneck process



The concurrent-cycles model

Micali G et al., Cell reports **25** (2018)
 Micali G et al., Science Advance **4** (2018)

Recapitulates all correlation patterns!

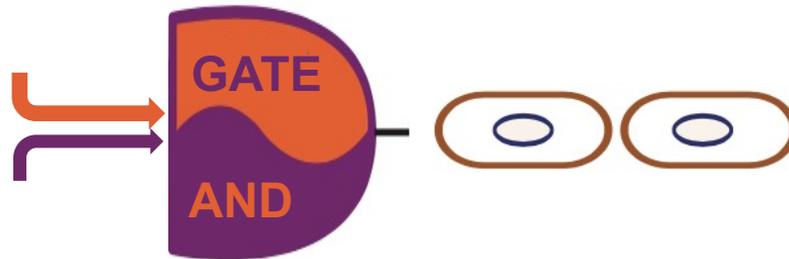


Recap on cell division coordination

1. **Replication is not rate-limiting for cell division**
(all models based on this assumption fail with data)

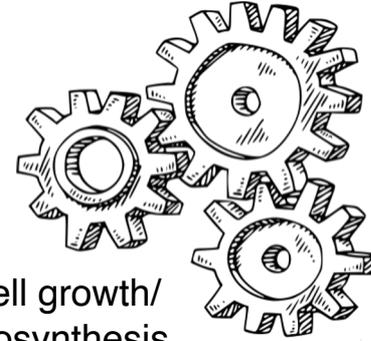
2. **“Chromosome agnostic” models**
(assuming that replication is never the bottleneck)
are also falsified by the data

3. **Concurrent processes set cell division**



Book chapter question (III)

Link with cell growth and biosynthesis? How do we go beyond studying correlation patterns?



Cell cycle progression

Cell growth/
biosynthesis

Other processes

Crosstalk between cell cycle control and cell growth/biosynthesis

(Sections 12.4)

Diana Serbanescu, Nikola Ojkic, and Shiladitya Banerjee. *The FEBS Journal*, 2021.

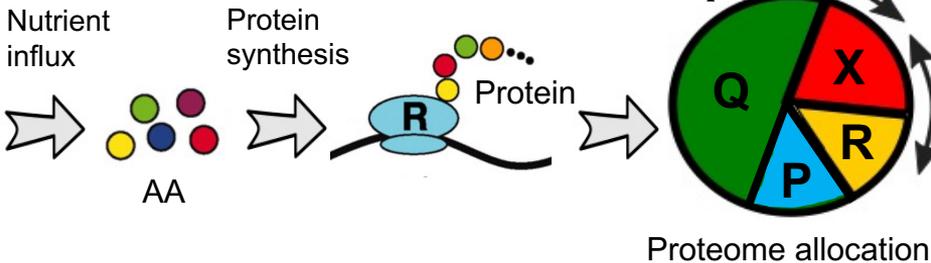
François Bertaux, Julius von Kugelgen, Samuel Marguerat, and Vahid Shahrezaei. *PLoS Computational Biology*, 16 (9), September 2020.

Parth Pratim Pandey, Harshant Singh, and Sanjay Jain. *Physical review. E*, 101:062406, June 2020.



Unified whole-cell coarse grained model

“Biosynthesis level”



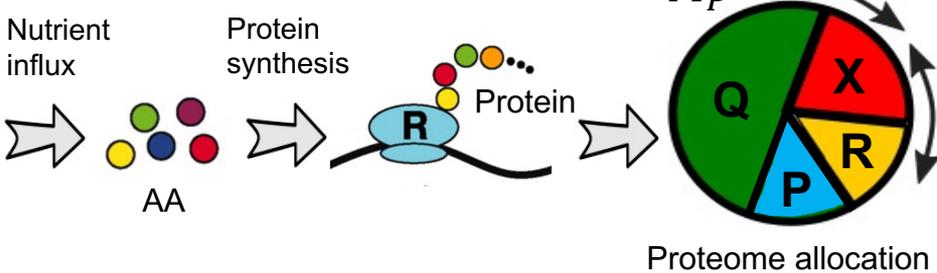
$$\frac{dA}{dt} = \frac{1}{m_A} (k_n P - a k_t R f_a + \sum d_{P_i} P_i)$$

$$\frac{dP_i}{dt} = \frac{1}{m_{P_i}} (a k_t f_{P_i} R f_a - d_{P_i} P_i)$$



Unified whole-cell coarse grained model

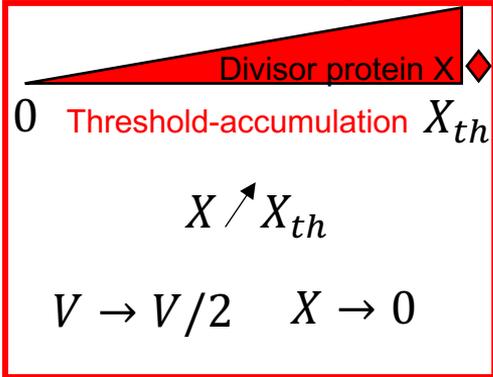
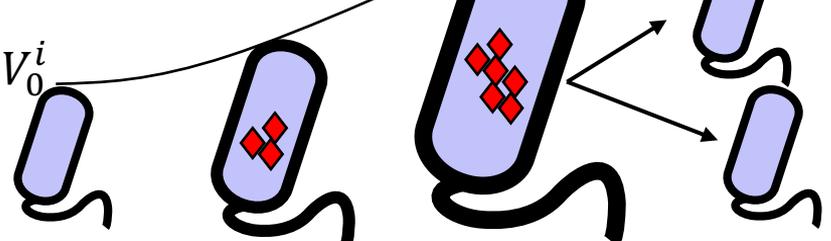
“Biosynthesis level”



$$\frac{dA}{dt} = \frac{1}{m_A} (k_n P - a k_t R f_a + \sum d_{P_i} P_i)$$

$$\frac{dP_i}{dt} = \frac{1}{m_{P_i}} (a k_t f_{P_i} R f_a - d_{P_i} P_i)$$

“Cellular level”



$$\frac{dV}{dt} = \lambda V$$

$$\frac{dX}{dt} = k_X V - \frac{d_X}{m_X} X$$

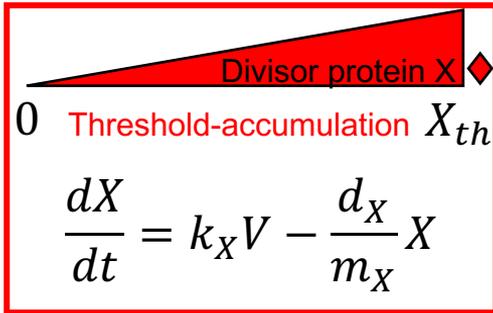


A molecular mechanism for the adder

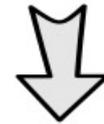
Molecular mechanism to obtain an adder (**threshold-accumulation**):

Division is set by the accumulation of a divisor protein N up to a threshold value.

1. Production rate proportional to volume
2. Target number of divisor proteins
3. Starting from zero



⇒
$$X(t) = \frac{k_X V_0}{\lambda + d_X/m_X} \left(2^{\frac{t}{\tau_d}} - 2^{-\frac{d_X}{m_X \lambda} t / \tau_d} \right)$$



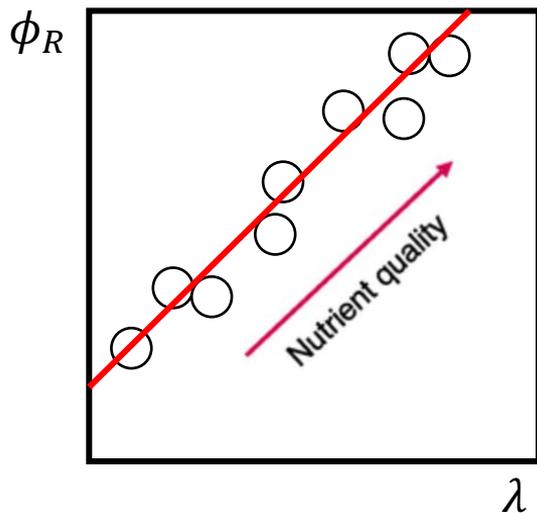
$\lambda \gg d_X/m_X$

$$\Delta V \approx X_{th} \frac{\lambda}{k_X} = \text{const.} \quad \text{Adder}$$



Growth laws and trade-offs between protein sectors

First growth law



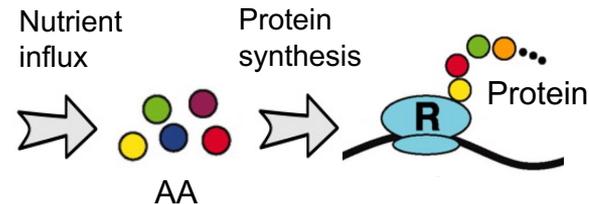
$$\lambda \propto (\phi_R - \phi_R^{min})$$

Derivation:

$$\lambda = \frac{1}{M} \frac{dM}{dt} = \frac{1}{M} \frac{dM_{prot}}{dt} + \frac{1}{M} \frac{dM_A}{dt} = \dots = \frac{k_n P(t)}{M}$$

At steady state fluxes are balanced:

$$\frac{k_n P^*}{M} = \frac{a k_t R^* f_a}{M}$$



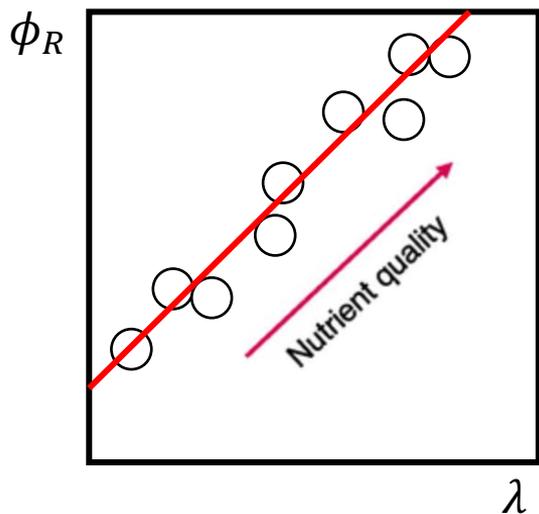
$$\lambda^* = \frac{a k_t M_P}{m_R M} (\phi_R - \phi_R^{inact})$$

Matthew Scott et al., Science 330,1099-1102(2010).



Growth laws and trade-offs between protein sectors

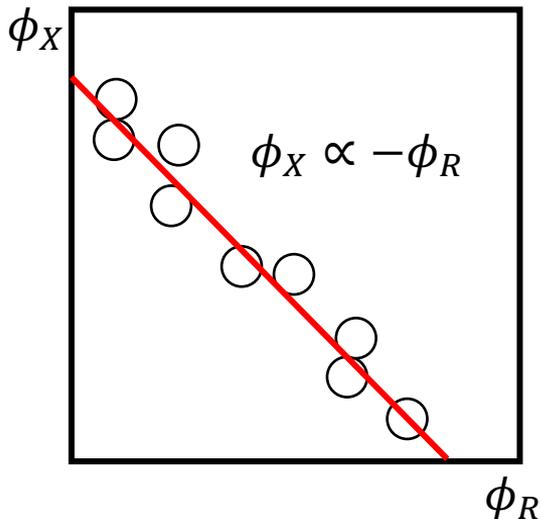
First growth law



$$\lambda \propto (\phi_R - \phi_R^{\min})$$

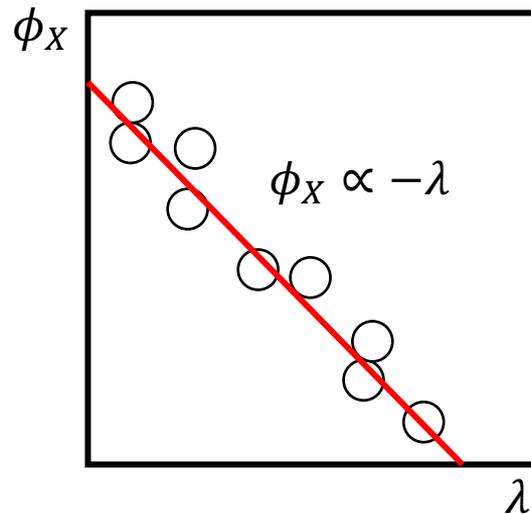
Matthew Scott et al., Science 330,1099-1102(2010).

Trade-offs between growth and division



$$\phi_X = -\frac{K_n + K_t}{K_n} \phi_R + \frac{K_t \phi_R^{\min} + K_n \phi_R^{\max}}{K_n}$$

Growth law for the division sector

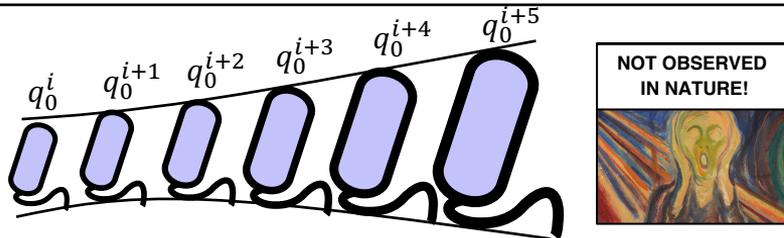


$$\lambda = \frac{K_n K_t}{K_n + K_t} \frac{M_{prot}}{M} (\phi_R^{\max} - \phi_R^{\min} - \phi_X),$$

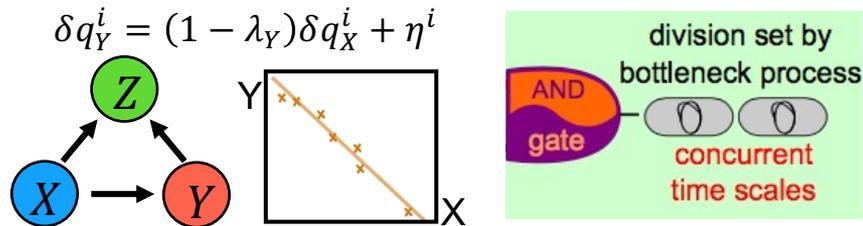


Take Home messages

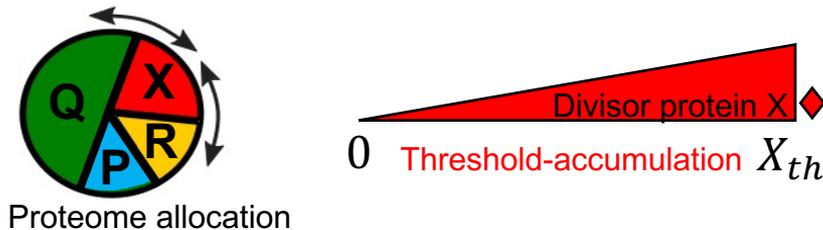
1. Cell division *per se* does not guarantee homeostasis (in *E.coli*).
Control mechanisms must be in place.



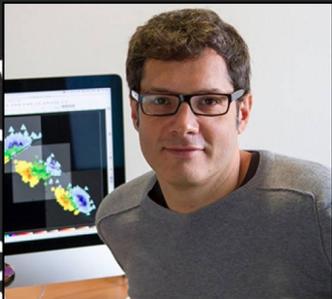
2. The Linear-response framework is helpful in making sense of single-cell correlation patterns and studying the decision to divide.



3. We are able to integrate gene-expression models with cell control strategies into a unified quantitative framework



I thank



Marco Cosentino
Lagomarsino, IFOM



Jacopo Grilli
ICTP



Gabriele Micali
Humanitas

...and you for the attention!

